16.6 #39

Area of The part of

$S: \; 3x+2y+z=6$

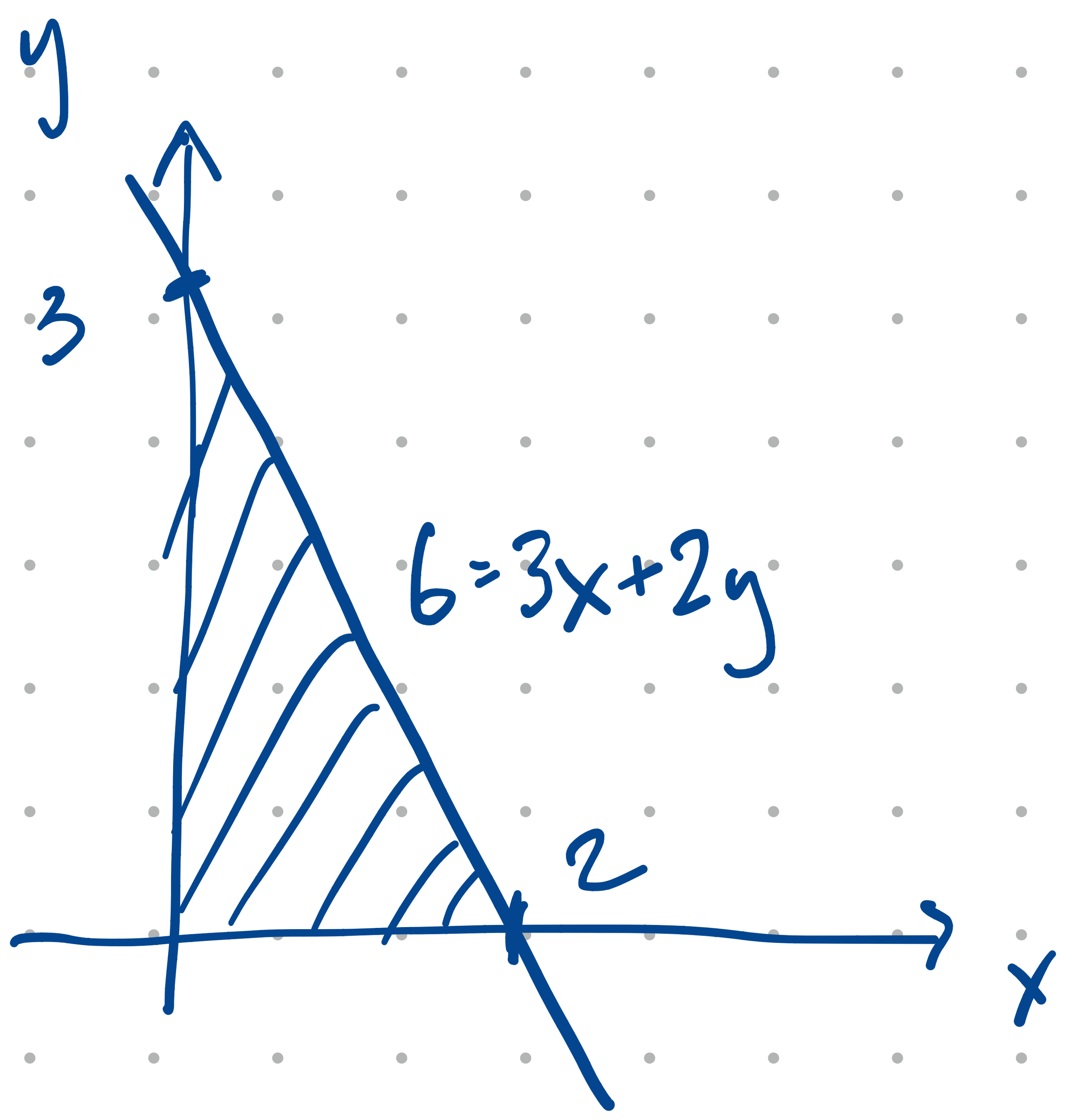
that lies "in the first octant."

Rmk. This is just a triangle, so this is very doable w/o calculus.

Solution. $z = 6-3x-2y$

parametrization:

$$\vec{r}(x,y) = \langle x, y, 6-3x-2y \rangle$$



"___" means $x \geq 0 \quad y \geq 0 \quad z \geq 0$

$$x \geq 0$$
$$y \geq 0$$
$$6-3x-2y \geq 0$$

So the integral for area:

$$\iint_S 1\,dS = \int_0^2 \int_0^{3-\frac{3}{2}x} 1 \quad |\vec{r}_x \times \vec{r}_y|\,dy\,dx$$

$$= \ldots$$

How to parametrize a surface given in Cartesian?

A: No general strategy. But if one var can be solved for in terms of the other two, then use the other two as parameters.

$$x^2 + y^2 + z^2 = 4$$

$$x = 2\sin\phi\cos\theta \qquad 0 \le \phi \le \pi$$

$$y = 2\sin\phi\sin\theta \qquad 0 \le \theta \le 2\pi$$

$$z = 2\cos\phi$$

Advice for matching/identifying parametric surfaces:
think about the grid curves
(hold one of u, v to be const.)

ex) $\langle u\cos v, u\sin v, v \rangle$ 16.6 #13

v const $\longrightarrow$ lines (through z-axis, perpendicular to it)

u const $\longrightarrow$ helixes

so picture IV.

16.6 #14: both families of grid curves are parabolas

so picture VI

From pollev:

1) $\underbrace{\text{div } (\underbrace{\text{div } \vec{F}}_{\text{scalar fn.}})}_{\text{nonsense}}$

2) $\vec{F} = \nabla f$ for some f

$\nabla \times \vec{F} = \nabla \times \nabla f = \vec{0}$

$\boxed{\text{curl of grad is } \overrightarrow{\text{zero}}}$

pf: use Clairaut's Thm.

3) $\vec{F} \times \vec{F} = \vec{0}$ (and its curl is then $\vec{0}$ as well.)

4) You can interpret vector calc. operations in $\mathbb{R}^2$, refer to p. 1108 – 1109 (end of §16.5).

5) $\boxed{\text{div of curl is zero}}$

Rmk: (in $\mathbb{R}^3$) $\quad \nabla =$ "$\left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle$"

This is not an actual "vector"

$$\nabla f = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle f$$

$$= \left\langle \frac{\partial}{\partial x} f, \frac{\partial}{\partial y} f, \frac{\partial}{\partial z} f \right\rangle$$

i.e. $\langle f_x, f_y, f_z \rangle$

$$\nabla \cdot \langle P, Q, R \rangle = \frac{\partial}{\partial x} P + \frac{\partial}{\partial y} Q + \frac{\partial}{\partial z} R$$

$$= P_x + Q_y + R_z.$$

6) I claim $\quad \nabla \cdot (\underbrace{\nabla f}_{\langle f_x, f_y, f_z \rangle} \times \underbrace{\nabla g}_{\langle g_x, g_y, g_z \rangle}) = 0$ (zero scalar fn)

Check by direct computation:

$$\nabla f \times \nabla g = \det \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ f_x & f_y & f_z \\ g_x & g_y & g_z \end{bmatrix}$$

$$= \langle f_y g_z - f_z g_y,\ f_z g_x - f_x g_z,\ f_x g_y - f_y g_x \rangle$$

$$\nabla \cdot (\nabla f \times \nabla g)$$

$$= \left(\cancel{f_{yx} g_z} + \cancel{f_y g_{zx}} - \cancel{f_{zx} g_y} - \cancel{f_z g_{yx}} \right)$$

$$+ \left(\cancel{f_{zy} g_x} + \cancel{f_z g_{xy}} - \cancel{f_{xy} g_z} - \cancel{f_x g_{zy}} \right)$$

$$+ \left(\cancel{f_{xz} g_y} + \cancel{f_x g_{yz}} - \cancel{f_{yz} g_x} - \cancel{f_y g_{xz}} \right)$$

where things cancel b/c of <u>Clairaut's thm</u>.